On Capacity Options in Lean Retailing

by

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Abstract

In this study, we investigate the strategy of increasing the production capacity temporarily through contingent contractual agreements with short-cycle manufacturers to manage the risks associated with demand uncertainty. We view all these agreements as capacity options. More specifically, we consider a simple model of a production system that supplies products to meet a random demand that switches randomly between a high level and a low level. The production system has enough capacity to meet the demand in the long run, but does not have enough capacity to meet the demand by production when the demand is high. Therefore, it is necessary to produce-to-stock in advance. Alternatively, a contractual agreement with a short-cycle manufacturer can be made. This option gives the right to receive additional production capacity when needed. There is a fixed cost to purchase this option for a period of time and if the option is exercised, there is an additional per unit exercise cost which corresponds to the cost of the goods produced at the short-cycle manufacturer. We formulate the problem as an optimal control problem and analyze it analytically. By comparing the costs between two cases where the agreement with the short-cycle manufacturer is used or not, we determine the price of this option. Furthermore, we investigate the effects of demand variability on this contract.

Key Words: Options, Capacity planning, Stochastic Modeling

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1. Introduction

The changes in the retailing industry and also in the customer demand are forcing the manufacturers in the supply chain to adopt new strategies to cope with the new environment. In the last decade, one of the major transformations in the retailing industry has been the emergence of the concept of lean retailing (Abernathy et al., 1999). As a result of diffusion of lean retailing practices, more and more retailers are using new approaches to distribution, forecasting, planning and organizing production, and coordinating the supply chain.

This study is motivated by the challenges in the retail-apparel-textile channel. The retail-apparel-textile channel is characterized by rapidly changing styles, uncertain customer demand, product proliferation, and long lead times.

In the apparel industry, the lead times from the retailer order to delivery are quite long. For example, the average lead times for Benetton and Liz Claiborne apparel are five months (Signorelli and Heskett, 1984; Dalby and Falherty, 1990), for oxford shirts ordered by J.C. Penney, it is 7 months (Skinner, 1992) and the lead time for Jaymar Ruby slacks is 8 months (Hammond, 1991).

As a result of these long lead times, the risks of having too little or too much inventory increases if the retailers have to place the orders long before the season. Reducing these lead times has been the primary objective in the apparel industry as a result of the Quick Response movement (Hammond, 1991). Moreover, it is shown that delaying production and ordering decisions to incorporate revised demand forecasts in the production schedules reduces these risks (Fisher et al., 1994; Fisher and Raman, 1996).

The studies on Quick Response shows that the savings of in-season replenishment may justify paying from 30%-50% more to a supplier that provides in-season replenishment (Hunter, et al., 1996; Pinnow and King, 1997). As a result, lean retailers demand from manufacturers to supply a higher percentage of its orders within a selling season.

In addition to long lead times, product proliferation is increasing the risks a manufacturer faces. As the product proliferation increases, the variability of demand for each item also increases. Faced with these challenges, a manufacturer should devise better strategies to respond quickly to changes in the demand.

From a manufacturer’s perspective, the ways to respond quickly to changes in demand are producing to stock in advance, increasing the production capacity permanently by investing in new production facilities, increasing the production capacity temporarily by using overtime,
subcontracting, etc., and combinations of these pure strategies. Making these decisions in the most effective way are of crucial importance for the manufacturer to be competitive.

One of the strategies is to incorporate the information on demand variability effectively in production, sourcing and inventory management decisions. Abernathy, et.al. (2000) show that making production and inventory decisions separately for product families grouped according to their variability improves the performance considerably. Furthermore, they show that making sourcing decisions based on the speed of delivery in addition to the cost is the best way to improve the performance. More specifically, using a faster supplier, probably a local short-cycle manufacturer, can be very beneficial, especially, for items with high variability. Additional costs of using the short-cycle manufacturer can easily be justified by reducing the inventory levels and its associated costs.

In this study, we investigate the strategy of increasing the production capacity temporarily by means of contingent contractual agreements with short-cycle manufacturers to manage the risks associated with demand uncertainty. We view all these agreements as real options. An option is the right, but not obligation, to take an action in the future and a real option is the extension of financial option theory on real (non-financial) assets (Amram and Kulatilaka, 1999). Since the contractual agreement is related to increasing the capacity, we refer this option as a capacity option.

More specifically, we consider a simple model of a production system that supplies products to meet a random demand that switches randomly between a high level and a low level. The production system has enough capacity to meet the demand in the long run, but does not have enough capacity to meet the demand by production when the demand is high. Therefore, it is necessary to produce-to-stock in advance. Alternatively, a contractual agreement with a short-cycle manufacturer can be made. This option gives the right to receive additional production capacity when needed. There is a fixed cost to purchase this option for a period of time and if the option is exercised, there is an additional per unit exercise cost which corresponds to the cost of the goods produced at the short-cycle manufacturer. At the beginning of the planning horizon, the manufacturer decides how much additional capacity will be reserved at the short-cycle manufacturer. Then at a given time, the manufacturer decides how much to produce and how much additional production to be requested from the short-cycle manufacturer.

We formulate the problem as an optimal control problem and analyze it analytically. By comparing the costs between two cases where the agreement with the short-cycle manufacturer is used or not, we determine the price of this option. Furthermore, we investigate the effects of demand variability on this contract.

The contribution of this study is two-fold. First, an analytical model is developed and analyzed thoroughly to investigate capacity options as a way to manage demand uncertainty in production systems. Second, the model is used in numerical experiments to gain some insight when these options are valuable.
Due to the simplicity of the model, the study is geared towards showing the direction of the benefits that can be obtained through options, rather than answering a more specific question of how a company can price such an option to use in day-to-day decision-making.

Organization of the remaining part of this paper is as follows: In §2, a review of the pertinent literature is given. The basic model and its assumptions are given in §3. The production control problem is formulated in §4, and solved in §5. The analysis of the option is provided in §6. Numerical results that investigate the effects of variability are presented in §7. Finally, the concluding remarks are given in §8.

2. Past Work

The studies reviewed on using capacity options are grouped in two areas: inventory management and stochastic modeling of manufacturing. Most of the work on capacity options are found in inventory literature where the main objective is to determine the order quantities.

Jain and Silver (1995) investigate a case where both the demand and also the capacity of the supplier are uncertain and the capacity of the supplier can be reserved by paying a premium. The replenishment quantity and the amount of capacity to reserve are determined to minimize the overall costs. Costa and Silver (1996) extend this approach to a multiperiod analysis.

In the apparel catalog industry, contracts similar to options are used. Eppen and Iyer (1997) present backup agreements used in apparel catalog industry. Under this agreement, the buyer makes a firm commitment to purchase a given number of goods, $Q$, at the beginning of the horizon. In the first period, the buyer purchases a certain percentage of this commitment, $\rho$, at a given price $c$. At the second period, if the buyer purchases less than the committed, a penalty of $b$ dollars per unit is paid for the remaining parts that are not purchased. It is reported that a catalog company Catco uses these contracts with Anne Klein and DKNY ($\rho=0.2$, $b=0$) and with Liz Claiborne ($\rho=0.25$, $b=0.2c$) (Eppen and Iyer, 1997).

Another contract type is the quantity flexibility contract (Bassok and Anupindi, 1997). Under quantity flexibility contract, the buyer provides a forecast of future orders to the supplier. Later, the buyer purchases between a predetermined minimum and the maximum levels within the initial forecasts. That is, a minimum quantity needs to be purchased at the agreed price and there is an option to purchase up to the maximum level at the same price. The buyer can update the forecasts later on. These contracts are reported to be used in the electronics industry, for example, by IBM printer division, Sun Microsystems, Solectron, Hewlett Packard, etc. (Anupindi and Bassok, 1998; Tsay and Lovejoy, 1999).

If the capacity is scarce, a buyer may pay an upfront fee to reserve capacity in advance. For example, in the semiconductor industry, according to a recent survey by the Fabless Semiconductor Association, 30% of capacity reservation arrangements are take-or-pay. However, agreements called pay-to-delay capacity reservation are also used (Brown and Lee, 1997). Under this agreement, the buyer makes an agreement with the supplier to purchase a minimum quantity at a given price $c_f$
and pays $c_0$ per unit to reserve up to a level. These additional units can be purchased at an extra unit cost of $c_e$.

In a recent study, Barnes-Schuster et. al. (2000) examine the role of options in a two-stage buyer-supplier system. Using a two-period model with correlated demand, they show that options provide flexibility to a buyer to respond to market changes in the second period. They also investigate the implications of such arrangements for coordination of the supply chain. They argue that return policies could be used that coordinate the channel and give the supplier positive profits. In this study, all three agreements discussed above are discussed as special cases of a general option.

Two main sources of uncertainty in a production-inventory system are supply and demand uncertainty. Since, the main focus in the inventory studies is on demand uncertainty, these models generally assume unlimited and always available production. Contrarily, in the stochastic modeling of manufacturing area, the main focus is on the supply side variability and it is assumed that the production capacity is limited and interrupted due to uncertain events such as machine failures.

The methodology employed in this study is similar to the ones in the stochastic modeling of manufacturing area. Gershwin (1994a) discusses how the production control of a manufacturing system can be formulated as a stochastic optimal control problem where the production rate is controlled in real-time to minimize the average inventory carrying and backlogging costs while the production system is prone to failures. In most of these studies, the main source of uncertainty is the occasional unavailability of the manufacturing facility, mainly due to machine failures, while the demand is assumed to be constant. Furthermore, the production capacity is limited.

Gershwin (1994b) introduces an extension of Flexible Manufacturing System (FMS) scheduling model where the capacity of the FMS can be increased, if necessary. For a long-term increase, more machines can be bought or more operators can be hired, while for a short-term increase, excess demand can be contracted or labor hours can be increased with overtime. He shows that the optimal policy is similar to an $(s,S)$ policy where subcontracting is used when the inventory position goes down to a lower hedging level and the parts are produced with the maximum rate until an upper hedging level is reached. A proof of optimality of the hedging point policy for this problem is given by (Huang, et. al., 1999). Both of these models assume that the demand is constant and the production rate of the machine and also the rate at which the contractor can supply goods are greater than the demand rate. Furthermore, the objective is to minimize the average inventory carrying, backlog costs and cost of increased capacity.

Tan (2000) extends the models where the demand is constant to piece-wise constant, random switching demand models. The random switching model can be used to approximate a given demand process with its variability. By transforming the system with uncertain demand to an equivalent system where the demand is constant, it is shown that the optimal policies for constant demand can be used to determine the optimal policies for uncertain demand.

In light of these reviews, the contribution of this study can be stated. The model presented in this study is an extension of the above-mentioned studies on stochastic models of manufacturing systems. First, it is assumed that the main uncertainty is in the demand side and the production
system is always available as opposed to the main assumption that the supply is uncertain and the
demand is constant in these studies. This extends the scope of these studies to handle demand
variability. Second, the main objective in this study is to maximize the profits whereas in the
previous studies the objective was cost minimization. Third, the model considers production
capacity constraint explicitly where the production capacity is assumed to be unlimited in most of
the inventory papers. The model also considers the case that the production capacity and the short-
cycle manufacturer’s capacity are less than or equal to the demand when it is high. Moreover, the
model is a long-range performance evaluation model as opposed to two-period models studied.
Finally, due to the modeling framework, a complete analytical solution that yields a closed-form
solution is possible.

3. Model Description

We consider a make-to-stock system with a single manufacturing facility that produces to
meet the demand for a single item. The product flow is approximated by a continuous flow. The
demand rate at time \( t \) is denoted by \( d(t) \). The state of the demand at time \( t \) is \( D(t) \) which is either
high (H) or low (L). When the demand is high, the demand rate is \( \mu_H \) and when the demand is low,
the demand rate is \( \mu_L \). At time \( t \), the amount of finished goods inventory is \( x(t) \).

The times to switch from a high demand state to a low demand state and from a low demand
state to a high demand state are assumed to be exponentially distributed random variables with
rates \( p \) and \( r \). Representation of demand in this way is similar to the demand for an item, which is
stationary in the long run, but the mean shifts temporarily as a result of promotions etc. However,
the time to switch is uncertain and cannot be easily predicted. Because of its memoryless property,
the exponential distribution provides a good approximation for this time. That is, analyzing the
time since the last state change does not change the expected time until the next state change.

Alternatively, it can be argued that there are numerous items, e.g. SKUs, and the total
required demand for these parts shifts between a high level and a low level. Furthermore, it is
shown that, the asymptotic distribution of the total demand generated by this random switching
model is normal (Tan, 1997). Tan (1997) gives the asymptotic variance rate of the total sojourn
time in an alternating renewal process with exponential switching times. Using this result, it can be
shown that the mean and variance of the amount of demand that will arrive per unit time, \( E[N(t)] \)
and \( \text{Var}[N(t)] \) given as

\[
\bar{d} = \lim_{t \to \infty} \frac{E[N(t)]}{t} = \mu_L + (\mu_H - \mu_L)e^{1-e^p}
\]  

(1)

\[
\sigma^2 = \lim_{t \to \infty} \frac{\text{Var}[N(t)]}{t} = 2(\mu_H - \mu_L)^2 \frac{1}{r}(1-e)^2
\]  

(2)
where \( e = \frac{r}{p + r} \) is the percentage of the time the demand is high. Now if \( \bar{d} \) and \( cv = \sigma / \bar{d} \) are given, the parameters \( e, p \) and \( r \) can be chosen by using the above equations.

The maximum production rate of the manufacturing facility is \( \mu \). The production rate of the manufacturing facility at time \( t \) is denoted by \( u(t) \). \( 0 \leq u(t) \leq \mu \). We assume that the production capacity is sufficient to meet the demand when it is low but insufficient when it is high, i.e., \( \mu_L < \mu < \mu_H \). However, it has enough capacity to meet the demand in the long run, i.e.,

\[
\mu > \bar{d} = \mu_H \frac{r}{p + r} + \mu_L \frac{p}{p + r}
\]

The profit generated through the sales of the goods produced at the plant is \( L \) (dollars per unit). The inventory carrying cost is \( c^+ \) and the backlog cost is \( c^- \) (dollars per unit per time).

**Option of Receiving an Additional Capacity When Needed**

Consider the following capacity option: the company pays an upfront fee of \( CO \) to a short-cycle manufacturer to receive an extra capacity of \( 0 \leq v(t) \leq \mu_L \) at time \( t \) for a period of \( T \). The exercise cost of the option, i.e., the production cost when it is obtained from the short-cycle manufacturer is above the regular production cost by \( \Delta c \) $/unit. After paying the additional cost, the profit generated through the sales of the goods received from the subcontractor is \( A \) (dollars per unit). Then \( \Delta c = L - A \).

Since it is uncertain when the demand will be high, the company may consider this option to decrease the need of holding an excessive inventory or investing in capacity expansion. This is also advantageous for the contractor if it has extra capacity not fulfilled with its own demand. Furthermore, the upfront payment will be received regardless of whether the option is exercised or not in the specified time period.

The maximum amount that can be paid for the upfront fee \( CO \) is the difference between the best profit that can be obtained by using the outside contractor and the one that is obtained without using the outside contractor in the contract period.

In this study, the effects of this kind of agreement on the performance of the production-inventory system are analyzed. We consider the profit, service level, and the average inventory as the main performance indicators.

### 4. Production Control Problem

At time \( t \), the manufacturing facility is scheduled to produce at rate \( u(t) \), and the subcontractor is requested to supply goods at the rate of \( v(t) \) in such a way that the expected profit is maximized.

The profit is the difference between the money generated through sales and the inventory carrying and backlog costs. Then the production control problem is
Problem 1:

\[
\text{Max } \Pi_1 = E \int_0^T \left( Lx(t) + A c^+ x^+ + c^- x^- \right) dt
\]

subject to

\[
\frac{dx(t)}{dt} = u(t) + v(t) - d(t)
\]

\[
0 \leq u(t) \leq \mu
\]

\[
0 \leq v(t) \leq \mu_c
\]

\[
d(t) = \begin{cases} 
\mu_H & \text{if } D(t) = H \\
\mu_L & \text{if } D(t) = L 
\end{cases}
\]

Markov dynamics for \(S(t)\) with rates \(p\) (from \(H\) to \(L\)) and \(r\) (from \(D\) to \(U\))

For implementation purposes, a feedback solution of this problem that yields \(u(t)\) and \(v(t)\) as a function of the state variable \(x(t)\) is desirable. It is known that a \((Z, S)\) hedging type policy is optimal to minimize the average inventory and backorder costs for an unreliable production system with constant demand (Gershwin, 1994; Hu, et. al., 1999).

Let us consider an equivalent system with a constant demand with rate \(\mu_H\) and an unreliable machine with two failure states up and down. The switching time from the up state to the down state, and from the down state to the up state, are exponential with rates \(r\) and \(p\) respectively. Furthermore, the maximum production rates in the up and down states are \(\mu + \mu_r\mu_b\) and \(\mu\) respectively. With this transformation, the sample path of this system will be equivalent to the system with constant production and uncertain demand.

Therefore the optimal policy for the constant production-uncertain demand problem is also a \((Z, S)\) policy where \(Z\geq 0\) is the produce-up-to level, and \(S\leq 0\) is the backlog level when it is reached, the subcontractor is requested to supply goods at a rate that keeps the backlog at this level and when the backlog is below this level, the subcontractor supplies with the maximum capacity. This policy drives the backlog/surplus into the region between \(Z\) and \(S\). If the subcontractor capacity is sufficient to keep the backlog at this level, i.e., \(\mu_c \geq \mu_H\), \(x(t)\) stays bounded between \(Z\) and \(S\) and \(x(t) < S\) is transient. If the subcontractor capacity is not sufficient, the lower hedging level cannot be sustained and \(S\) acts as a switching point below which the subcontractor is requested to supply goods with its maximum rate. However, even with the maximum subcontracting, the backlog level decreases due to insufficient capacity but at a lower rate of decrease. In this study, the case where the subcontractor has sufficient capacity to keep the backlog at a given level, i.e., \(\mu_c \geq \mu_H\) is analyzed in detail. The case of insufficient capacity is discussed in the numerical results section.
5. Analysis of the Model

Figure 1 shows sample realizations for the cases where the subcontracting is not used and the option is used and not used. Figure 2 depicts the cumulative production and demand for these cases.

The performance analysis of the model is carried by determining the differential equations that explain the behavior of the system in the interior states and solving these equations subject to some boundary conditions.

When the amount of finished goods inventory is not equal to the hedging level and it is not zero, the system is said to be in the interior region. In this region, the system state at time $t$, $S(t)$, is expressed by a tuple $(D(t), X(t))$ where $D(t) \in \{H, L\}$ and $S \leq x(t) \leq Z$.

The time dependent system state probabilities for the interior region, $F_i(t,x)$ is defined as

$$F_i(t,x) = P[D(t) = i, \ x(t) \leq x] \quad i \in \{H, L\}, \ t \geq 0, \ S < x < Z. \quad (8)$$

The time-dependent system state density functions are also defined as:

$$f_i(t,x) = \frac{\partial F_i(t,x)}{\partial x} \quad i \in \{H, L\}, \ t \geq 0, \ S < x < Z. \quad (9)$$

Figure 1. The effect of the option on the surplus/backlog.

$\mu = 1, \ \mu_H = 1.5, \ \mu_L = 0.5, \ \bar{d} = 0.9, \ \overline{cv} = 3, \ \epsilon = 0.08, \ \epsilon^f = 0.24, \ L-A=9.$
Figure 2. The effect of the option on the cumulative demand and production. \( \mu = 1, \mu_h = 1.5, \mu_l = 0.5, \bar{d} = 0.9, cv = 3, c = 0.08, c^+ = 0.24, L-A=9. \)
We assume that the process is ergodic and, thus, the steady-state density functions exist. The steady-state density functions are defined as:

\[ f_i(x) = \lim_{t \to \infty} f_i(t, x) \quad i \in \{H, L\}, \quad t \geq 0, \quad S < x < Z. \]  

(10)

The probabilities that the finished goods inventory is equal to the produce-up-to level and \( S, P_Z \) and \( P_S \) are given as

\[ P_Z = \lim_{t \to \infty} P[x(t) = Z] \]

(11)

\[ P_S = \lim_{t \to \infty} P[x(t) = S] \]

(12)

We first consider the behavior of the process in the interior regions \( S < x < Z \). Conditioning the probability density of the event \( \{(D(t+\delta t), X(t+\delta t)) = (H,x)\} \) on the state of the system at time \( t \) yields

\[ f_H(t+\delta t, x) = f_H(t, x-(\mu - \mu_H)\delta t)(1-p\delta t) + f_L(t, x-(\mu - \mu_L)\delta t)(r\delta t) + o(\delta t) \]

where \( o(\delta t) \) approaches to zero faster than \( \delta t \). The above equation can be written in differential form for \( \delta t \to 0 \) as

\[ \frac{\partial f_H(t, x)}{\partial t} + (\mu - \mu_H)\frac{\partial f_H(t, x)}{\partial x} = -pf_H(t, x) + rf_L(t, x) \]

(14)

Taking the limit of the above equation as \( t \to \infty \) yields the following differential equation for \( f_H \)

\[ (\mu - \mu_H)\frac{df_H(x)}{dx} = -pf_H(x) + rf_L(x) \]

(15)

Following the same steps for \( f_L \) yields

\[ (\mu - \mu_L)\frac{df_L(x)}{dx} = pf_H(x) - rf_L(x) \]

(16)

Note that, since \( \mu_L < \mu < \mu_H \), the inventory level decreases when the demand is high and increases when demand is low.

In order to solve the set of first order differential equations given in (15) and (16), two boundary conditions are needed. First, note that at any given level of the finished goods inventory, the number of upward crossings must be equal to the number of downward crossings. Let \( N(i, \xi, T) \) denote the total number of level crossings in state \( i \), at surplus level \( \xi \), in the time interval \( [t, t+T] \) for large \( T \). Then
The renewal analysis shows that
\[
\lim_{T \to \infty} \frac{N(i, \xi, T)}{T} = \Delta r f_i(\xi)
\]
(18)
where \(\Delta r\) is the rate of change in the buffer level at state \(i\), and \(f_i(x)\) is the steady-state density function (Yeralan and Tan, 1997). Then, equation (17) can be written as
\[
(\mu_H - \mu)f_H(x) = (\mu - \mu_L)f_L(x).
\]
(19)
Using this result in equation (15) gives the following first order differential equation
\[
\frac{df_H(x)}{dx} = \left(\frac{p}{\mu_H - \mu} - \frac{r}{\mu - \mu_L}\right)f_H(x)
\]
(20)
whose solution is
\[
f_H(x) = c \cdot e^{\lambda x}
\]
(21)
where \(\lambda = \frac{p}{\mu_H - \mu} - \frac{r}{\mu - \mu_L}\) and \(c\) is a constant to be determined. Following equation (19),
\[
f_L(x) = c \frac{\mu_H - \mu}{\mu - \mu_L} e^{\lambda x}
\]
(22)

Now consider the probabilities that the finished goods inventory is equal to the produce-up-to level; \(P_Z\). Since \(\mu_L < \mu < \mu_H\), the inventory level can reach this level only when the demand is low, so the manufacturing capacity can add to the finished goods buffer. Each time the inventory level increases and reaches the level \(Z\), it stays there for a period until the state of the demand changes to high and the inventory level starts decreasing. This time is equal to \(1/r\). By the ergodicity argument, \(P_Z\) is equal to the time average of the total sojourn in this state:
\[
P_Z = \lim_{T \to \infty} \frac{N(L, Z, T)}{T} \frac{1}{r} = (\mu - \mu_L)f_L(Z) \frac{1}{r} = c \frac{\mu_H - \mu}{r} e^{\lambda x}
\]
(23)
Similarly
\[
P_S = \lim_{T \to \infty} \frac{N(H, S, T)}{T} \frac{1}{r} = (\mu_H - \mu)f_H(S) \frac{1}{p} = c \frac{\mu_H - \mu}{p} e^{\lambda S}
\]
(24)
Finally, the constant \(c\) is determined by using the normalizing condition. Namely, the sum of all the probabilities must add up to 1:
Using equations (21), (22), (23), and (24) with the normalizing condition given above yields

\[
c = \begin{cases} 
\left( e^{\lambda z} \left( \frac{\mu_H - \mu_L}{\lambda (\mu - \mu_L)} + \frac{\mu_H - \mu}{r} \right) - e^{\lambda S} \left( \frac{\mu_H - \mu_L}{\lambda (\mu - \mu_L)} - \frac{\mu_H - \mu}{p} \right) \right) \lambda \neq 0 \\
\left( Z - S \right) \frac{\mu_H - \mu_L}{\mu - \mu_L} + \left( \frac{\mu_H - \mu}{p} + \frac{r}{p} \right) \lambda = 0
\end{cases}
\]

(26)

Now, since the complete solution of the system is available in the derived density functions and the probability masses, the performance measures of interest can be determined. One of the most important performance measures is the total sales per time \( TH \). Since backlog is allowed, \( TH \) is equal to the average demand rate.

The average finished goods inventory \( WIP \) is

\[
WIP = \int_0^Z [x f_H(x) + x f_L(x)] dx + Z \cdot P_Z
\]

(27)

which is evaluated as

\[
WIP = \begin{cases} 
\left( e^{\lambda z} \left( \frac{\mu_H - \mu_L}{\lambda (\mu - \mu_L)} + \frac{\mu_H - \mu}{r} \right) - e^{\lambda S} \left( \frac{\mu_H - \mu_L}{\lambda (\mu - \mu_L)} - \frac{\mu_H - \mu}{p} \right) \right) \lambda \neq 0 \\
c \cdot \left( Z^2 \frac{\mu_H - \mu_L}{2(\mu - \mu_L)} + Z \frac{\mu_H - \mu}{r} \right) \lambda = 0
\end{cases}
\]

(28)

Similarly, the average backlog level \( BG \) is

\[
BG = E[x^-] = -\int_S^0 [x f_H(x) + x f_L(x)] dx - S \cdot P_S
\]

(29)

which is evaluated as

\[
BG = \begin{cases} 
\left( e^{\lambda S} \left( \frac{\mu_H - \mu_L}{\lambda (\mu - \mu_L)} - \frac{\mu_H - \mu}{r} \right) - e^{\lambda S} \left( \frac{\mu_H - \mu_L}{\lambda (\mu - \mu_L)} + \frac{\mu_H - \mu_L}{\lambda^2 (\mu - \mu_L)} \right) \right) \lambda \neq 0 \\
c \cdot \left( S^2 \frac{\mu_H - \mu_L}{2(\mu - \mu_L)} - S \frac{\mu_H - \mu}{r} \right) \lambda = 0
\end{cases}
\]

(30)

The average rate at which the subcontractor supplies goods is

\[
TH_v = (\mu_H - \mu) P_S = c (\mu_H - \mu)(\mu - \mu_L) e^{\lambda S}
\]

(31)

Finally the profit is
\[ \Pi_1 = L \cdot TH \cdot (L - A) \cdot TH_p - c^+ \text{WIP} - c^- \text{BG} \quad (32) \]

Pricing the Option

Once the profits with and without the option are determined excluding the fixed cost, the maximum amount that can be paid for this option for a given time interval can be calculated from the difference. Note that the calculated profits \( \Pi_1^* \) and \( \Pi_2^* \) are per unit time. Let \( T \) be the expiration time of the option. The expected profit in \([0, T]\) can be approximated with \( \Pi_1^* T \) and \( \Pi_2^* T \) as \( T \) approaches infinity. Then the amount that can be paid for this option should not exceed the total additional profit obtained from this option:

\[ C_0 \leq (\Pi_2^* - \Pi_1^*) T \quad (33) \]

6. Numerical Results

In this section, we present some preliminary results. Figure 3 shows how the terms of the contract can be evaluated by using the relationship between the additional cost, gain in the profit, and the fixed cost and the duration of the option. For example, the figure shows that, it is possible to increase the profits by paying 20% of the expected profit without using the subcontractor during the duration of the option as an upfront payment (option price) and paying less than 120% of the regular production cost as you receive goods from the subcontractor (exercise price of the option).

Figure 4 illustrates the effects of demand variability on a specific option with an upfront payment of 20% of the profit without subcontracting and an exercise price which is 50% above the production cost. As the demand variability, summarized with its coefficient of variation, increases the value of the option increases. However, for low variability cases \( cv < 0.9 \), it is not profitable to use the option.

Figures 5 and 6 depict the case of insufficient subcontractor capacity. Namely, if the maximum rate at which the subcontractor can supply goods is now sufficient to meet the demand when it is high, i.e., \( \mu > \mu + \mu_c \), it is not possible to keep the backlog level at the lower hedging point \( S \). In this case, \( S \) is a switching point. When the demand is high, the backlog decreases with rate \( \mu H - \mu \) above \( S \), decreases with a reduced rate of \( \mu H - \mu - \mu_c \) below \( S \). This case is also analyzed by following a similar methodology.

In the cases, two subcontractors, one with sufficient capacity and another with insufficient capacity are considered. The additional production cost of the insufficient subcontractor is lower than the cost of the sufficient contractor. Both of these cases are also compared to the case no subcontractor is used. For illustrative purposes, the fixed cost of the options with the subcontractors are set to zero.
Figure 3. Evaluating the terms of the contract: fixed payment and additional production cost

Figure 4. Effects of demand variability on the Additional Profit ($\mu = 1, \mu_H = 1.5, \mu_L = 0.8, \bar{d} = 0.9, cv = 1, c = 0.3, c^+ = 0.1, L = 5, A = 2, C_0 = 20\% \Pi_T$)
Figure 5. Effects of demand variability on the cost and the profit ($\mu = 1$, $\mu_L = 2.5$, $\mu_U = 0.5$, $\mu_L = 8$, $A = 3$, $A' = 6$, $\mu_c = 1.5$, $\mu'_c = 0.45$, $C_0 = 0$)

Figure 6. Effects of subcontractor capacity on the cost and the profit ($\mu = 1$, $\mu_L = 2.5$, $\mu_U = 0.5$, $\mu_L = 8$, $A = 3$, $A' = 6$, $\mu_c = 1.5$, $\mu'_c = \rho(\mu_H - \mu)$, $C_0 = 0$)
Figure 5 shows the effect of demand uncertainty on the cost and profit for these three cases. When the demand variability is low, all these cases yield very close results. As the demand uncertainty increases, the option both with the sufficient and insufficient subcontractors give better results than the case where no subcontractor is used. The results for the sufficient and insufficient subcontractors are very close to each other. The insufficient subcontractor yields better results due to its lower additional production cost when the demand uncertainty is high.

Figures 6 shows the effects of the maximum capacity of the insufficient subcontractor on the profit and cost. The maximum production rate of the subcontractor is set to $\rho(\mu_H-\mu)$. When $\rho<1$, the subcontractor has insufficient capacity and when $\rho=1$, it has sufficient capacity to meet the demand when it is high. The additional production cost is the same for all cases. As the subcontractor can provide goods at a higher rate with the same additional production cost, the cost decreases and the profit increases. Furthermore, if the maximum capacity is lower than a specific level (where the curves for the insufficient and sufficient subcontractor cases intersect), it is more advantageous to use the subcontractor with higher capacity and higher additional production cost.

7. Concluding Remarks

The simple model considered in this study shows that option-type contractual agreements can be used as a strategy to cope with demand variability. The model shows that the value of this strategy increases as the demand uncertainty increases. This result supports the options view that uncertainty creates opportunity. Furthermore the additional costs of sourcing from a short-cycle manufacturer can be justified through increased sales and reduced finished goods inventories. However, this strategy is more advantageous for the producer. Subcontractors should have enough incentives to take part in such agreements. The upfront payment of the option may provide such an incentive. In this case, valuation of the option and deciding how to use this valuation in decision making are important questions that need to be answered.

The simplifying assumptions of the model makes it harder to use the results immediately in a corporate setting to determine the price of an option in a daily operation. This requires a more detailed model of the demand, production schedule, etc. However a simulation model and a simulation-based optimization method, such as the ordinal optimization, can be used for this purpose. Even in this case, determining the critical parameters of the model, especially, demand uncertainty and the demand levels is a challenge.

In addition to analyzing a producer with a single subcontractor, the framework can be extended to include competition among subcontractors and also among producers. Furthermore, the approach can be extended to investigate capacity expansion decisions of the producers and the investment decisions in short-cycle manufacturing. Thee extensions are left for future research.
References


